Practical 2 4600
State Space Controller Design

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Contents

1 Task

The plant consists of a permanent magnet dc electric servo motor/gearbox with angular position feedback. Open loop test have shown that the relationship between applied voltage (volts) and resultant rotational speed (rad/sec) can be described by the transfer function.

$$G(s) = \frac{1}{s+16} \tag{1}$$

The controller which should be designed has to meet the following performance criteria:

- Steady state error to a step change in desired position = zero
- Maximum percentage overshoot to a step change in desired position = 4.4
- Maximum settling time allowed following a step change in desired position = 0.25sec

The output of the Plants Transfer function is in (rad/sec) but the motor gives only an angular posiition as feedback therefore the Transferfunction (equation 1) must be Integrated. That means in Laplace domain it must be multiplicated with $\frac{1}{s}$.

$$G(s) = \frac{1}{s(s+16)} = \frac{1}{s^2+16s} \tag{2}$$

2 conventional P or PI contoller

Transfer function of the P controller: K. This results in an open-loop Transfer Function of the system

$$G_{open-loop}(s) = \frac{K}{s^2 + 16s} \tag{3}$$

From which its possible to derive the Closed Loop transfer function for the system

$$G_{closed-loop}(s) = \frac{G_{open-loop}}{1 + G_{open-loop}} = \frac{K}{s^2 + 16s + K} \tag{4}$$

This is a type 1 system.

$$G = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{5}$$

The percentage overshoot for a stop input is than

$$\begin{aligned} P.O. &= 100e^{-\pi\frac{\zeta}{\sqrt{1-\zeta^2}}} \\ \zeta &= \frac{-\ln(\frac{P.O.}{100})}{\sqrt{\pi^2 + \ln(\frac{P.O.}{100})^2}} \\ \zeta &= 0.705 \\ 16 &= 2\omega_n\zeta \\ \frac{8}{\zeta} &= \omega_n = 11.346 \end{aligned} \tag{6}$$

to meet the Maximum settling time of 0.25sec

$$T_s = \frac{4}{\zeta\omega_n} \rightarrow \omega_n = \frac{4}{T_s\zeta} = 22.693$$

But if ω_n would be great enough to meet the settling time criteria it would no longler meet the overshoot criteria for the P controller.

In order to meet the settling time criteria the system should have a pole location with a real part of -16 but on this place, as the root locus plot shows (figure 1), is it with a P Controller only possible to get pole location with an imaginary part of zero. But a imaginary part of grater than zero is needed to meet the overshoot criteria.

Even if an PI controller is chosen (an real pole was added in the root locus plot), the system becomes first unstable till an additional real zero was added (figure 2), too. But even if the real zero is moved it is not possible to find a pole location with an Real part of < 16 and imaginary part of > 0.

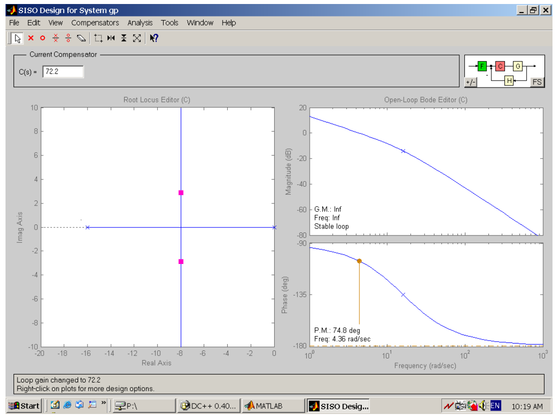

Figure 1: Rootl Locus Plot for P Controller

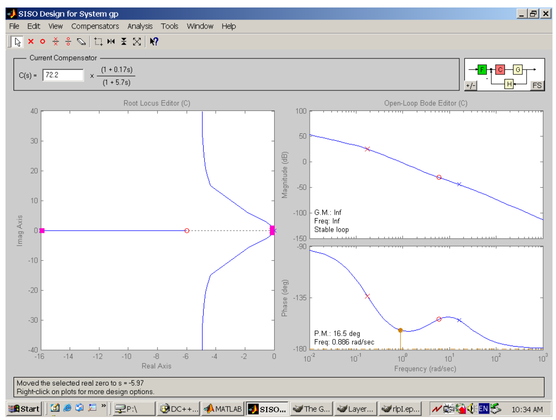

Figure 2: Rootl Locus Plot for PI Controller

3 State space model

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s^2 + 16s} \tag{7}$$

$$Y(s)s^2 + Y(s)16s = U(s) \tag{8}$$

in the time domain

$$\ddot{y} + 16\dot{y}(t) = u(t) \tag{9}$$

States will are defined as follwing:

$$\begin{aligned} y_1(t) &= y(t) \rightarrow \dot{y_1}(t) = \dot{y}(t) = y_2 && (10) \\ y_2(t) &= \dot{y}(t) \rightarrow \dot{y_2} = \ddot{y}(t) && (11) \end{aligned}$$

with the states (eq 11) equation 9 can be written as

$$\dot{y_2} + 16y_2(t) = u(t) \tag{12}$$
$$\dot{y_2} = -16y_2(t) + u(t) \tag{13}$$

In matrix form

$$\begin{bmatrix} \dot{y_1}(t) \\ \dot{y_2}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -16 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{14}$$
$$y(t) = [1\ 0] \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} \tag{15}$$

This gives the following results for the system matrixes

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -16 \end{bmatrix} \tag{16}$$
$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{17}$$
$$C = [1\ 0] \tag{18}$$

To check the system for controllability

$$CM = [A\ AB] = \begin{bmatrix} 0 & 1 \\ 1 & -16 \end{bmatrix} \rightarrow det \neq 0$$

that means the system is controllable.

Desired Pole locations are $-16 \pm 16.1j$.

The Gain Matrix F can be found with $det\left((sI - (A - BF))\right)$ and where derived as $F = [515.21 \quad -48]$