SP3005/PH3025 ADVANCED BIOMECHANICS
Laboratory 2
Rotational Motion: Moment of Inertial and Conservation of Angular Momentum

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Contents

1 Introduction

The reluctance of a body to move linearly is a measure of its mass, m, which is a constant parameter for any given body. A body also exhibits a reluctance to be turned or rotated about an axis; this property of the body is called moment of inertia, I. Once a body is in some state of linear motion it will remain in that state unless acted upon by an external force. This is a statement of the law of conservation of momentum. similarly a body in some state of rotation will remain in that state unless acted upon by an external torque. This is the law of conservation of angular momentum. In this practical we investigate both moment of inertia and angular momentum.

2 Theory

Consider a sphere of mass m attached to one end of a massless spindle. The other end of the spindle is fixed to an axis which is free to rotate. The distance from the axis to the centre of the sphere is r. Suppose the sphere is rotating about the axis with angular acceleration α.

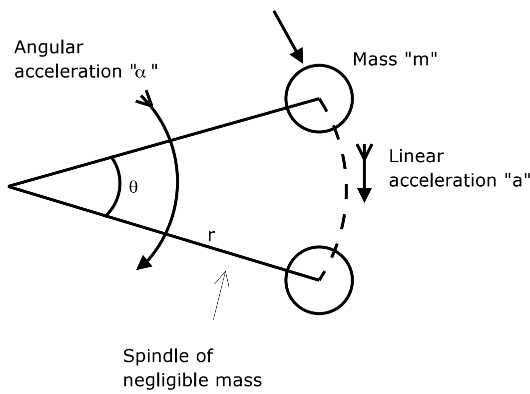


Figure 1: A rotating sphere

For the linear aspect of its motion we have

$$F = ma \tag{1}$$

The magnitude of the torque produced by the force F causing the rotation is given by

$$\tau = rF$$

that is,

$$\tau = rma$$

Now in terms of the angular acceleration, the magnitude of the linear acceleration is given by

$$a = r\alpha$$

Therfore

$$\tau = mr^2\alpha$$

This can be written as

$$\tau = I\alpha \tag{2}$$

We identify the product mr^2 with reluctance of the sphere to rotate, i.e., with its *moment of inertia* I. In other words the moment of inertia of a small sphere rotating at a distance r about an external axis is given by

$$I_{sphere} = mr^2$$

Thus the reluctance of the sphere to rotate depend not only on its mass, but also on its distance from the axis. In general the reluctance of a body to rotate depends not only on its mass but also on the distribution of its mass about the axis of rotation, and we write

$$I = mk^2$$

where k is called the radius of gyration an it is a measure of the mass distribution of the body about the axis of rotation. The *radius of gyration* is derived variable that expresses the radial distance from the axis of ration at which the mass of the segment can be concentrated without altering the moment of inertia of the segment.

2.1 Angular momentum

An object of mass m moving linearly with velocity v possesses a linear momentum of p given by $p = mv$ An object of moment of inertial I roationg with angular velocity ω possess angular momentum L given by,

$$\begin{aligned} L &= I\omega \\ &= mk^2\omega \end{aligned}$$

Note that both ω and L are vector quantities. The magnitude of ω is the rate of change of the angular position θ. The direction of ω is define such that, with the fingers of the right hand curled in the direction of rotation, the thumb points along ω. Thus an object lrotating anticlockwise in the x-y plane about an axis parallel to the z axis has angular velocity

$$\omega = \omega\hat{k} = (0, 0, \omega)$$

and angular momentum

$$\begin{aligned} L &= I\omega\hat{k} &= (0, 0, I\omega) \\ &= mk^2\omega\hat{k} &= (0, 0, mk^2\omega) \end{aligned}$$

where $\hat{k}$ is a unit vector in the z direction.

In the absence of any net external torque the angular momentum of a system is conserved, i.e.,

$$L = \text{constant vector}$$

or

$$\triangle L = 0$$

2.2 Angular impulse

To angularly accelerate a body, i.e., to change its angular momentum, a net external torque must be applied. The change in angular momentum depends upon both the magnitude of the torque and the time the torque is applied for. Specifically

$$\tau \triangle t = \triangle L = I(\omega_2 - \omega 1)$$

where

$$\begin{aligned} \tau &= \text{external torque} \\ \triangle t &= \text{time during which } \tau \text{ is applied} \\ \omega_1 &= \text{angular velocity at start of } \triangle t \\ \omega_2 &= \text{angular velocity at end of } \triangle t \end{aligned}$$

3 Aims

The aims of this practical are to:

1. develop a "feel" for angular motion
2. investigate how the moment of inertia of the human body about an axis changes when the mass distribution about that axis changes.
3. test the law of conservation of angular momentum.

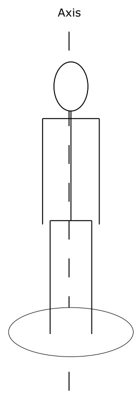
4 Equipment

- Angular-momentum table
- Weights
- Stopwatch
- Camcorder
- Gyroscope
- Lighting

5 Procedure

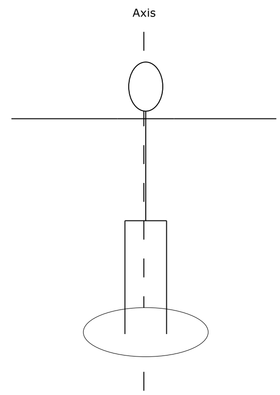
5.1 Moment of Inertia

The diagram below shows three positions for the rotation of the human body about a vertical axis



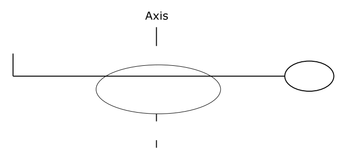
I_1

Figure 2: Position 1



$I_2 = 2I_1$

Figure 3: Position 2



$I_3 = 10I_1$

Figure 4: Position 3

1. Select a suject and have them stand in position 1(figure 2) on the angular-momentum table
2. A member of the group should then grip the outer edge the table and firmly, but gently, rotate the table back and forth.
3. The remaining members of the group should each do this is turn.
4. Have the subject now change to position 2 (figure 3)
5. Repeat steps 2 & 3 for the new position
   Note down your observation with the change of position.
6. Have the subject change to position 3(figure 4), and repeat steps 2 & 3 for this position.
   Note down your observation with change of position.
7. Replace the subject with another member of the group and allow the initial subject to go through the above procedure.

Explain your observation in view of the theory presented in section 2. Repeat the above exercise for position 1 and 2 with subject holding a dumbbell in each hand.
Do you "feel" any difference in the change of position from 1 to 2 with the dumbbells than without them? Explain.

5.2 Angular Momentum

1. Have a subject stand motionless on the table holding the 'bat' inn both hands. Swing the bat overhead in large circles. Note your observations.

2. Perform step 1 without the bat and compare.

3. Perform the following arm actions while standing motionless on the table.

   (i) Abduct the right arm to shoulder level, and flex the left arm to shoulder level. Swing both arms vigourously the left. Note your observation.

   (ii) Repeat step (i) with a dumbbell in each hand. Compare with step (i).

   (iii) Stand motionless on the table and abduct both arms to shoulder level. Simultaneously horizontal abduct both arms. Note your observation.

   (iv) Repeat (iii) holding a dumbell in the right hand. Note your observation.

Explain your observations in steps 1,2 and 3 in terms of the theory presented on page 1.

5.3 More Angular Momentum

1. Have a subject stand motionless on the table.

2. "Wind up" the gyroscope.

3. Hand the gyroscope to the subject with its angular momentum vector pointing in the x direction.

4. Instruct the subject to tilt the gyroscope such that L points up. Not your observation.

5. Instruct the subject to tilt the gyroscope such that L points down. Note your observation.

Explain you observations in steps 4 and 5 in terms of the theory presented on page 1.

5.4 Determination of Moment of Inertial

1. Have a subject stand in position 1 on the table with a dumbbell in each hand.
2. Start the table rotating.
3. Start the camcord.
4. Measure the angular velocity of the table.
5. Instruct the subject to abduct his/her arms to position 2.
6. Measure the angular velocity of the table.

Using conservation of angular momentum determine the relative size of the two moments of inertia. Compare your answers using the stop watch with that from the video analysis.

6 Results and Discussion

6.1 Moment of Inertia

After the subject changed to position 2 is was harder to change the velocity of the table. It became even harder when he changed to position 3. By changing from position 1 to 2 to 3 the subject increased it's moment of inertia. After equation 2 an bigger moment of inertia will result in an increased torque needed to change the current velocity of the subject.

If the subject takes the dumbbell in each hands the torque needed to change his angular velocity will increase again. This assumption was affirmed by the experiment.

6.2 Angular Momentum

When the subject starts swinging the bat he start rotating in the same direction he swings the bat. Without the bat instead he does not start rotating at all. When he swings the bat he produces an angular momentum the systems try's to stay in an equilibrium. The moving of the bat disturbs this equilibrium. Looked from a single point of the perimeter the disturbance is the bat leaving. Therefore the system will produce an angular momentum which is directed against the disturbance. This results in spinning in the same direction the bat is swung.

After he swings the arms to the left the table starts rotating to the right with dumbbells in each hand he spins quite more than without. When he swings to the left the table swings to the right to stay in equilibrium the effect is more significant with dumbbells because than the angular momentum produced by the swing in the first place is much grater.

On experiment (iii) and (iv) no movement could be observed. Because he moved in the wrong plane the produced angular momentum was in the wrong axis to result in an movement.

6.3 More Angular Momentum

When the gyroscope is handed to the subject the angular momentum vector L points in x direction. Because the subject can not rotate in the x direction he will stand still till he starts to tilt the gyroscope. When the gyroscope is rotated in such a manner that L points up he start to rotate clockwise.

The cause of this spin is the angular momentum of the gyroscope, because of the gyroscope rotating anti clockwise the system try's to achieve an equilibrium therefore it must start rotating clockwise to compensate the angular momentum of the gyroscope. If L points down he starts spinning anti clockwise as expected.

6.4 Determination of Moment of Inertial

$$L = I\omega$$

$$L = I_1\omega_1 \qquad L = I_2\omega_2$$

$$I_2 = X * I_1$$

$$I_1\omega_1 = L = X * I_1\omega_2$$

$$\underline{\frac{\omega_1}{\omega_2} = X = \frac{t_2}{t_1}} \tag{3}$$

position	camcorder t_{90° $(frames)$	stop watch t_{3*360° (sec)
1	41	5.79
2 - abducted arms	62	16.78
X (eq. 3)	1.51	2.90

Table 1: angular velocity

The change of position of the subject results in an increase of the moment of inertia of the person. I_2 is 1.51/2.90 times bigger than I_1.

7 Conclusion

Because of the last experiments big differences of the values of t measured with the stop watch and measured with the video no precise answer can be given about which amount I_2 is bigger than I_1. But for sure I_2 is at least 50% bigger than I_1.

All other experiments show how easy the body's moment of inertia can be changed with just changing the position. This for example, is used by figure skaters when they pirouette. They use their arms or legs to increase or decrease the spinning speed.